A.4 (NMPA) is a consequence of (NMPP), (Refl), and (ASym)

	(1) $x < y$	assumption for conditional proof
1	(2) $\neg y < x \vee x = y$	from (ASym), 1
1	(3) $(x < y \wedge \neg y < x) \vee x = y$	from 1, 2
1	(4) $x \ll y \vee x = y$	from (NMPP), 3
	(5) $x < y \rightarrow (x \ll y \vee x = y)$	conditional proof, 4, discharging 1
	(6) $x = y \rightarrow x < y$	from (Refl)
	(7) $x \ll y \rightarrow x < y$	from (NMPP)
	(8) $(x \ll y \wedge x = y) \rightarrow x < y$	from 6, 7
	(9) $x < y \leftrightarrow (x \ll y \vee x = y)$	from 5, 8

A.5 (PPEq) is a consequence of (NIPP), (NMPP), and (ASym)

	(1) $x \lneq y$	assumption for conditional proof
1	(2) $x < y \wedge y = x$	from (NIPP), 1
1	(3) $\neg y < x$	from (ASym), 2
1	(4) $x < y \wedge \neg y < x$	from 2, 3
1	(5) $x \ll y$	from (NMPP), 4
	(6) $x \lneq y \rightarrow x \ll y$	conditional proof, 5, discharging 1
	(7) $x \ll y$	assumption for conditional proof
7	(8) $x < y \wedge \neg y < x$	from (NMPP), 7
7	(9) $x = y$	Leibniz's law, 8
7	(10) $x < y \wedge x = y$	from 8, 9
7	(11) $x \lneq y$	from (NIPP), 10
	(12) $x \ll y \rightarrow x \lneq y$	conditional proof, 11, discharging 7
	(13) $x \ll y \leftrightarrow x \lneq y$	from 6, 12

2

Parthood is Identity[1]

KRIS MCDANIEL

1. Motivation and Initial Formulation

There is a strange feature of contemporary philosophical discourse and practice, one that is especially prominent in metaphysics. Contemporary philosophical inquiry is by and large driven by intuitions, and moreover (and obviously so) by the intuitions of those privy to the current conversation. Perhaps these intuitions are converging on the truth, but one can't help but worry, at least a little, that we live in a degenerate age in which the intuitions of the select few are either on non-convergent trajectories or are converging towards the False. A study of the history of philosophy can make very vivid the possibility of a philosophical community converging towards a set of views that many now find indefensible.

A related point is that philosophy is as subject to fads as anything else. When I was an undergraduate, one of the hottest topics in contemporary m&e—it has become customary, for reasons obscure to me, to lump metaphysics and epistemology under the heading 'm&e'—was vagueness. People lived and breathed vagueness. Nowadays, not as much, but almost certainly this is not because all the problems involved with vagueness have been solved. People by and large have just moved on. Part of the explanation for the 'moving on' is that, as problems receive more attention, the literature on the problem necessarily becomes more intricate, detailed, and complicated, and so more difficult to follow, let alone evaluate. And so the pool of people attending to a topic will shrink over time. I will confess to having

[1] Thanks to Andre Gallois, Cody Gilmore, and Joshua Spencer and audiences at the University of Auburn and the University of Geneva for extremely helpful comments on earlier drafts.

long given up trying to follow the literature on the metaphysics of causation for just this reason. This is by no means a disparaging remark about the metaphysics of causation; I merely cite myself as an example of someone susceptible to this 'philosophical drift'.

As the pool of people addressing a set of issues thereby contracts over time, the odds increase that some views will fall out of favor due to lack of consideration rather than compelling refutation. For these reasons, I think metaphysicians have an intellectual duty to periodically reconsider unfashionable theories to see if they have applications in new contexts. Views that are out of fashion might be merely out of fashion, and might surprise us with their hidden resources once they are brought back to contend with the puzzles of the day. My project here is to examine an older, currently unfashionable view—the view that the relation of numerical identity always obtains relative to some index, such as a time or a region of spacetime—and see whether it has the resources to solve some current extant issues in the metaphysics of material objects.

Let *strong composition as identity* be the view that (i) numerical identity is a non-distributive relation that can relate one thing to many things and (ii) a whole is numerically identical to its parts.[2] (A one–many relation is non-distributive just in case it is not distributive; a one–many relation R is distributive just in case whenever one thing bears R to some things, it bears R to each of those things.) In what follows, I will make a case for a stronger view than strong composition as identity: the slogan of this view is that *parthood is identity*. Informally, the view I will defend is that (i) a whole is numerically identical to *each* of its parts considered *individually* rather than collectively and (ii) parthood can be analyzed in terms of identity. Since a view like this seems initially highly implausible, to say the least, it is surprising that a case can be made for it, provided that certain reasonable to believe background assumptions are granted.

I will assume without argument the following theses. I do not regard these assumptions as eliminable; in fact, I don't see how the view that parthood is identity can be defended without them. (Perhaps this fact will provide comfort to the foes of these theses.) Most of these theses I not only assume but think more likely to be true than not. The thesis that identity

[2] See Baxter (1988a) for a defense of strong composition as identity, and Lewis (1991) and Sider (2007) for defenses of a "moderate" form of composition as identity.

is relative to an index is the thesis that I am offering up for consideration, both my own and yours.

First, I assume that objects persist through time by enduring rather than perduring: things lack temporal parts, but rather enjoy full bodily presence at each moment they occupy. I assume that enduring objects can survive changes in both their parts and their properties.

Second, I assume a spacetime framework according to which the notion of a time is not fundamental, but rather is to be defined in terms of the notion of a region of spacetime. Given that spacetime regions are ontologically prior to times, the endurantist should claim that, strictly speaking, an enduring object is wholly present at different regions of spacetime.[3]

Third, I assume that enduring objects undergo change by having different properties at different regions of spacetime. Here I plump for an 'adverbialist' version of endurantism, according to which the relation of instantiation that links an object to a property, or some objects to a relation, is always relative to some region or other: an object instantiates-at-R a property F.[4] Every property or relation instantiated by some enduring object is instantiated relative to some region or other. That said, it may be that for some object O and property F, O instantiates F relative to each region that O occupies. In these situations, we will call F a *permanent* property of O. If it is the case that all objects that have F at some region have F permanently, then call F a *permanent* property simpliciter.

Similarly, there might be some objects O1 and O2 and some relation H such that, O1 and O2 instantiate H relative to some region, and moreover there is no region at which O1 and O2 instantiate some relation without instantiating H. In these situations, say that H is a *transported* relation of O1 and O2. If for every pair of objects that bear H at some region, H is a transported relation of those pair, then call H a *transported* relation simpliciter. (I hope it is clear how the notion of a transported relation can be generalized to cover relations of adicity greater than two. However, since we won't need the more general notion here, we won't pause to formulate it.)

[3] The view that enduring objects endure across regions of spacetime is briefly defended in van Inwagen (1990b, p. 4) and McDaniel (2004); it is extensively explored in Gilmore (2006).
[4] See Haslanger (2003) and (1989) for a discussion of the adverbialist view. Note that on her formulation of adverbialism, the adverbialist is neutral on whether there is a relation of instantiation. I would prefer to be neutral as well, but for the purposes of this chapter, I will speak more committedly. However, this commitment is eliminable.

These three assumptions are shared by many endurantists. The fourth assumption is less widely shared: I will provisionally assume a version of the thesis that identities can be 'temporary'.[5] *Every* property or relation enjoyed by enduring objects is had relative to a region. It is consistent with this claim that identity is a transported relation simpliciter, and perhaps this is the default position. I assume, however, that identity can fail to be a transported relation: x and y might be identical relative to some region R but not identical relative to some other region R′. (And at R′ they will instantiate some relation other than identity.)[6]

One reason to embrace 'temporary' identity is that it solves puzzles arising from fission and fusion.[7] The left half of a worm is crushed by a boot. The worm is mutilated but endures. So a worm can survive the loss of half of its body. What if it had been the right half that had been crushed? The worm would have been mutilated but would have endured. Again, the worm can survive the loss of half of its body. Suppose we bisect a worm with a surgical knife. Two non-identical worms, Lefty and Righty, are the result. Which is the original worm? Let t′ be the time of bisection, and t some time shortly prior. It seems that, at t, Lefty is identical with the original worm, but so is Righty. And so at t, Lefty is identical with Righty. But it seems that at t′, Lefty is not identical with Righty. According to the doctrine of temporary identity, things are exactly as they seem. (It is worthwhile to remember that many of our students find this response initially very attractive when they first consider the puzzle of fission.) Similar remarks apply to puzzles in which two things fuse into one. Since we have adopted the spacetime framework, strictly the thing to say is that things that are not identical at one region can be identical at another region.

Does this version of temporary identity conflict with the principle that identity is transitive? Not if this principle is properly formulated! Since identity is relative to a region, the proper formulation of the transitivity of identity

[5] The classic defenses of temporary identity are Gallois (1998) and Myro (1997).

[6] The question of what we should say about the numerical identity and distinctness of non-spatiotemporally located objects, such as platonic numbers or pure sets, is an interesting and pressing question. Perhaps such entities are self-identical at every region; or perhaps they are self-identical at some other, non-spatiotemporal, index; or, more radically still, perhaps the way in which they exemplify properties and relations is not the same way that material objects exemplify properties or relations. Each of these options is worthy of serious consideration. Unfortunately, I lack the space to tackle these questions here.

[7] See Gallois (2005).

is this: if x = y relative to R, and y = z relative to R, then x = z relative to R. Note also that the doctrine of temporary identity is consistent with the fact that the property of being self-identical is permanent simpliciter.

Anyone who says something non-standard about identity needs to say something non-standard about the law of the indiscernibility of identicals. As standardly formulated, the law states that if x = y, then every property of x is a property of y (and vice versa). But, as is well known, this simple formulation leads to trouble very quickly if we allow temporary identity. A natural reformulation of this law is the following. We will need to distinguish between what I will call *region-encoding* and *region-free* properties and relations. A region-encoding property is one that has information about either particular regions or regions in general built into the property. Some examples of region-encoding properties include *being an occupant of region R, being tall relative to R, being red at some region, being identical to x at R*, etc. Although all properties and relations are instantiated relative to a region, not all properties and relations are region-encoding. Some region-encoding relations are relations to regions. It is one thing to instantiate a property or a relation relative to that region; it is another altogether different thing to participate in a relation, one of whose relata is itself a region. Some examples of region-free properties include *being tall, being identical to x*, and *being red*. We will now formulate this law of the indiscernibility of identicals as follows: If x is identical to y at R, then x has some region-free property at R if and only if y has that region-free property at R.[8]

We've noted that there is something to be said for the doctrine of temporary identity. We should think even more favorably of this doctrine if it plays an unexpected role in solving extant philosophical problems, resolves thorny questions, or provides new and intriguing analyses of familiar notions.

It is clear that there is some intimate relation between a whole and its parts that is not exemplified by distinct objects. Consider a now stock example.[9] The farmer owns a farm composed of six plots of land, each of which he sells to different individuals. It would not be reasonable for the farmer to then sell the farm to a seventh individual. In some sense, the farm is nothing over and above its six plots. For this reason, some philosophers have embraced the slogan that composition just is identity: the intimate relation

[8] This way of 'restricting' Leibniz's Law is inspired by Myro's (1997) proposal. Koslicki (2008, pp. 47–69), argues that restricting Leibniz's Law in this fashion is a 'suspect strategy'. I don't have space to respond to Koslicki's interesting argument here.

[9] We owe this example to Baxter (1988a, p. 579); it is further discussed by Lewis (1991, pp. 83–84).

between a whole and its parts is the relation of many–one identity. You can't get more intimate than identity.

There are two worries about the view that composition is identity. The first worry is conceptual or semantical: isn't the relation of identity obviously one–one rather than one–many? Don't attempts to express the doctrine fail to even be grammatically well formed? ("They are it" is arguably not a grammatical English sentence.)[10] The second worry is metaphysical. If composition is identity, then in some sense the whole must be a duplicate of its parts (since it is the parts). So fix the properties and relations of the parts, and you thereby fix the properties of the whole. In short, if composition is identity, then the properties of a whole supervene on the properties of its parts. But some have alleged that 'emergent properties', i.e., properties that do not supervene on the properties and relations of their parts, are possible and perhaps even actual.[11]

Although these worries are decisive neither individually nor collectively, they do provide a motivation to search for an alternate explanation of the intimacy of the relation of part-to-whole. The view that parthood is identity is such an explanation. On this view, a whole is identical to each of its parts. (It is better to say that, for each part, the whole is identical to that part at some region. Which region? More on that momentarily.) This view is not committed to the claim that identity is fundamentally a many–one relation: the only primitive notions it employs are the ordinary notions of identity, parthood, and relative-to-a-region-instantiation. These notions are ones that the endurantist arguably needs anyway. Moreover, as well we see in section 2, one of these notions, specifically parthood, can be analyzed in terms of identity given this theory.

Additionally, that parthood is identity is consistent with the claim that a whole has features that do not supervene on the properties and relations of its parts. In general, the endurantist does not believe that the properties an object has at one region are metaphysically determined by the properties that an object has at different regions. (The properties that I have after changing from one state to another may well be *causally* determined by the properties I had at earlier states, but they are not *metaphysically* determined

[10] This worry is pushed by van Inwagen (1994). I'm not sure how much weight should be put on the alleged ungrammaticality of sentences like "They are it; it is them." But I doubt it is very much. However, the intuition that identity is a one–one relation deserves to be taken into consideration.

[11] This worry is pushed in McDaniel (2008).

by them.) The phenomenon here is in principle no different: although I might be (at some region or regions) identical to my parts (considered individually) it does not follow that the properties my parts have at some region are determined by the properties they have at other regions.

Finally, one might worry that composition is identity is inconsistent with the conjunction of endurantism and the obvious fact that objects gain and lose parts as they persist through time. Probably this worry is why friends of composition of identity tend to also be foes of endurantism. But there is an obvious way for the friend of composition as identity to endorse endurantism: hold that some identities are temporary.[12] Although I am these things *herenow*, I am not them *therethen*. On pain of embracing absurdity, the endurantist who embraces the view that composition is identity is committed to temporary identities just as much as the friend of the view that parthood is identity.

The view that parthood is identity captures the intuition that the relation between part and whole is especially intimate at least as well as the view that composition is identity, and yet it does not face the worries noted above that face the view that composition is identity. (It might face its own worries, and whether this is so will be explored in section 4.) Let us explore it further.

2. Formal Reformulation

I informally characterized the view that parthood is identity as the conjunction of two theses: (i) a whole is numerically identical to *each* of its parts considered *individually* rather than collectively and (ii) parthood can be analyzed in terms of identity. We will now replace this informal characterization with a formal one. There are two tasks to complete. Given the assumptions elaborated in section 1, (i) needs to be understood as saying that, for each part of a whole, there is some region at which the whole is that part. The first task is to determine which region that is. Once we have completed this task, we will be able to provide a formal analysis of parthood in terms of identity.

All relations are instantiated relative to some region. We will proceed by in general identifying which kinds of regions are suitable to serve as the locus for the instantiation of a two-place relation.

[12] See Merricks (1999).

Think for a minute as an endurantist who has not yet embraced the spacetime picture and so who continues to think of instantiation as relative to a time. At t1, an object O1 in spatial region SR1 stands in relation H to an object in spatial region SR2. At t2, H does not relate these two objects. Switch now to the spacetime picture. Since we are endurantists, we hold that an object can be wholly present at different regions of spacetime. So although O1 occupies R1 (a spacetime region roughly corresponding to how we conceived of t1 and SR1), there are other regions besides R1 occupied by O1. Similarly, although O2 occupies R2 (a spacetime region roughly corresponding to how we conceived of t1 and SR2), there are other regions besides R2 occupied by O2. But let's set these other regions aside, and consider which region H is instantiated relative to. The obvious candidates are (i) R1, (ii) R2, (iii) both R1 and R2, (iv) the intersection of R1 and R2 (if it exists), (v) the union of R1 and R2, (vi) some other region that overlaps neither R1 nor R2, (vii) some region that merely overlaps each of R1 and R2. Possibilities (i) and (ii) strike me as unduly arbitrary, and of course for that reason possibilities (vi) and (vii) should also be set aside. Many occupied regions do not overlap, and for that reason, it is not true in general that dyadic relations are instantiated relative to the intersection of some regions occupied by the relata of the relation. So set aside (iv). There is no arbitrariness in holding that a dyadic relation is instantiated relative to two different locations, but this option does seem to me to involve a curious duplication of facts, and this tells against option (iii). (On option (iii), O1 and O2 instantiate H relative to R1 and they instantiate H relative to R2.) So although none of these considerations is conclusive, they do make possibility (v) the most plausible answer.

Some interesting consequences follow from (v). Suppose there is some region R at which O1 is a part of O2. Given (v), R is occupied by O2. Suppose the relation H alluded to above is the relation of parthood. In this case, intuitively R1 must be a subregion of R2. (Recall the slogan that 'the part is smaller than the whole', which is perhaps dubious when considering set-theoretic oddities, but is eminently plausible when restricted to enduring physical objects.) If R1 is a subregion of R2, then the union of R1 and R2 just is R2. The union of R1 and R2 is the region at which parthood is instantiated, and so R2 is the region at which parthood is instantiated and R2 is occupied by O2. In general, every region at which something is a part of a whole is a region occupied by that whole. A similar and shorter argument can be given for the claim that, if O1 is identical with O2 at R, then both O1 and O2 occupy R.

Consider some regions R1 and R2 that correspond roughly to, respectively, a region of space occupied by your left hand and a region of space occupied by yourself, at a time in which your left hand is a part of you. Revert now to the spacetime picture: I say that your left hand is a part of you at R2. (There are other regions presumably at which it is a part of you, since you have hopefully managed to persist through time while keeping track of your left hand.) On the view that parthood is identity, something stronger is true: you are identical to your left hand at R2. More generally, whenever O1 is a part of O2 at some R, O1 is identical with O2 at R. Note that it does not follow from this claim that you are identical with your hand at R1. (In fact, your hand is not even a part of you at R1.) Recall that identities can be 'temporary'.

We have now completed the first task involved in explaining the view that parthood is identity. Let us turn to the second task, that of providing an analysis of parthood in terms of identity. Let's start by noting that the following analysis won't do:

> PI-1: x is a part of y at R =df. x and y are identical at R.

Although PI-1 is the analysis that most fits the slogan 'parthood is identity', it won't fly. For although my hand is part of me at R (a region that I occupy), I am not a part of my hand at R, despite the fact that I am my hand at R. We need a more sophisticated analysis, one that breaks the symmetry of identity.

Fortunately, we have already noted an interesting asymmetry. Although my hand might be a part of me at a region occupied by me, my hand is not a part of me at a region occupied by my hand. The following proposal makes use of that asymmetry:

> PI-2: x is, at R, a part of y =df. x is, at R, identical with y; there is some region R′ such that (i) x is, at R′, identical with x; (ii) x is not, at R′, identical with y, and (iii) R′ is a proper subregion of R.

PI-2, unlike PI-1, does not imply that I am a part of my hand at a region I occupy. This is the result that we wanted. That PI-2 lacks this implication, however, is not immediately obvious. In fact, it might seem that the following argument shows that PI-2 does imply that I am a part of my hand at some region that I occupy: Consider some x and y such that x is a part of y at R. Given PI-2, x is identical with y at R. And since x is identical with y at R, every feature that x has at R, y has at R, and vice versa. Y has the feature *having x as a part* at R. So x has this feature as well. So x is a part of x at R. For the same reasons, y is a part of y at R. And since y = x at R, y is a part of x at R. So I am a part of my hand at R. Absurd!

Although this argument looks initially compelling, it is actually an invalid argument. Because 'identity can be temporary', we endorsed a version of Leibniz's Law that licenses one to move from (i) x is y at R and (ii) x is F at R to the conclusion that (iii) y is F at R *only if* F is a region-free property. (Recall that a region-free property is one that does not encode information about either particular regions or regions-in-general.) But having x as a part is not a region-free property given PI-2, for PI-2 partly analyzes facts about parthood in terms of facts about identity at regions. And so the argument is invalid. We are not entitled to infer either that x is a part of x at R or that y is a part of y at R.

Note that this argument is in fact an instance of the same kind of attempted reductio against temporary identity: assume that x is y at R1 but not at R2. x at R1 has the property of being identical with x at R2. So y at R1 has the property of being identical with x at R2. Contradiction! Any friend of temporary identity must reject any indiscernibility principle that implies that this is a valid argument.[13]

Some metaphysicians might wonder what notion of parthood is being defined by PI-2. For reasons of technical convenience, some metaphysicians work with a conception of parthood according to which everything is a part of itself. They therefore distinguish between what they call 'proper' and 'improper' parts of a whole: the whole is an improper part of itself. They might ask: is PI-2 meant to be an analysis of proper or improper parthood?

The conception these metaphysicians operate with strikes me as highly artificial, and I see no reason to ape it by defining parthood in such a way as to distinguish between 'improper' and 'proper' parthood. I am talking about parthood. That said, it will emerge that PI-2 implies that parthood has many logical features in common with how these metaphysicians have conceived

[13] Gallois (1998) proposes a different way of confronting arguments of this sort.

of proper parthood. (For example, given PI-2, parthood is transitive and irreflexive.)

How significant is it that we can provide an *analysis* of parthood? A theory is made more elegant when some subset of the concepts that were previously taken as unanalyzed are successfully analyzed via some of the remaining members of that set. And the analysis itself might make surprising predictions or help settle extant disputes. Does PI-2 do either of these things?

3. Further Motivations

Yes. First, if PI-2 is true, then there is a straightforward explanation why the relation of parthood is transitive. Although a few metaphysicians have denied that parthood is transitive, the vast majority of metaphysicians accept that it is transitive, and some have even claimed that it is constitutive of the concept of parthood that it is transitive.[14] An analysis of parthood that can show that parthood is transitive is thereby worthy of some respect.

In short, given PI-2, the transitivity of the parthood relation falls out of the transitivity of identity. Given the spacetime framework, the proper way to formulate the transitivity of parthood is this: For all regions R and objects x, y, and z, if x is a part of y at R, and y is a part of z at R, then x is a part of z at R. We will prove the transitivity of parthood via reductio:

1. x is a part of y at R, y is a part of z at R, but ~(x is a part of z at R) [assumption]
2. x is identical to y at R and there is some region R* such that (i) x = x at R*, (ii) ~(x = y at R*), and (iii) R* is a subregion of R. [from PI-2, premise 1]
3. y is identical to z at R and there is some region R** such that (i) y = y at R**, (ii) ~x = y at R**, and (iii) R** is a subregion of R. [from PI-2, premise 1]
4. Either x is not identical to z at R or ~ (there is some region R*** such that (i) z = z at R***, (ii) ~x = z at R***, and (iii) and R*** is a subregion of R) [from PI-2, premise 1]

[14] For example, Koslicki (2008, p. 257) suggests that were one to give up the transitivity of a putative parthood relation, one would thereby raise serious doubts about whether that relation is a parthood relation. See Varzi (2006) and Simons (1987) for a defense of the claim that parthood is transitive; Rescher (1955) contains the classic complaint against the transitivity of parthood.

5. It is not the case that x is not identical to z at R. [transitivity of identity, premises 2 and 3]
6. So there is no region R*** such that (x = x at R***, ~x = z at R*** and R*** is a subregion of R).
7. But there is a region R*** such that (x = x at R***, ~x = z at R*** and R*** is a subregion of R). [see below]
8. So parthood is transitive. [premises 1, 6, 7]

The only undefended premise is 7. 7 is supported by the following line of reasoning. First, since z is a composite object at R, R must have at least one proper subregion. I assume here that occupying a point-sized region suffices for being simple at that region. This is a contentious assumption, but one that arguably follows from PI-2 as well. (More on this in section 4.)

One of these proper subregions is a region at which x is located. A necessary condition on y's being a part of z is that y occupy a proper subregion of R. One of the subregions of R occupied by y is also a subregion at which x is part of y. Call this subregion T. A necessary condition on x's being a part of y at T is that x occupies a subregion of T.

Call this proper subregion S. S is a subregion of R, since *being a subregion of* is a transitive relation. x is identical with itself at every region that it occupies, and so x is identical with itself at S. However, z does not occupy S. (A putative part of z (at R) occupies S, but z is not identical with that part at S.) Since z does not occupy S, z is not identical with anything at S. (Recall the result shown earlier, that an object is identical with something at a region only if that object occupies that region.) So z is not identical with x at S. So there is a region R''' (namely S) such that (i) x = x at R''' (aka S), (ii) ~x = z at R''' (S again), and (iii) R''' (still S) is a subregion of R. This is premise 7. Reductio complete. Given PI-2, we can give an explicit proof the transitivity of parthood that explains *why* parthood is transitive. This is an extraordinary explanatory gain.

Given PI-2, further interesting logical properties of parthood can be determined. Parthood is provably irreflexive. To say that parthood is irreflexive is to say that there is no R and x such that x is a part of x at R. Proof via reductio: suppose at some R, x is a part of x. Then there is some subregion R' of R such that ~x = x at R'. This is absurd in itself, but we are also entitled to infer that at R', x = x. Reductio complete.

Parthood is provably anti-symmetric. (The proof is easy.) Parthood is anti-symmetric just in case, for any R, x, and y, if x is a part of y at R and y is a part of x at R, then x = y at R. A direct proof: suppose x is a part of y at R and y is a part of x at R. Then, given PI-2, x is identical with y at R.

Interestingly, although parthood is provably irreflexive and anti-symmetric, that parthood is asymmetric does not directly follow from PI-2 alone. This is a surprising result, and it is worthwhile to see why this is the case. Consider the following attempt to prove asymmetry. Suppose for reductio that at some region R, x is a part of y at R and y is a part of x at R. Then by anti-symmetry x = y at R. Since x = y at R and x = x at R, and x is a part of y at R, by Leibniz's Law, x is a part of x at R. But, by irreflexivity, x is not a part of x at R. So there is no region R and objects x and y such that x is a part of y at R and y is a part of x at R. So parthood is asymmetric.

The problem with this argument is that the restricted version of Leibniz's Law does not license the move from x's being a part of y at R to x's being a part of x at R, despite the fact that x = y at R. We've already seen an instance of this same mistake: since parthood is a region-encoding property, one is not entitled to employ Leibniz's Law in this fashion.

I find this result a little disquieting. Fortunately, Cody Gilmore has suggested to me the following modification to PI-2, which does imply that parthood is asymmetric.

> PI-3: x at R is a part of y =df. (a) x is identical with y at R; (b) there is some region R* such that (i) x is identical with x at R*, (ii) x is not identical with y at R*, and (iii) R* is a proper subregion of R; and (c) it is not the case that there is a region R** such that (i) y is identical with y at R**, (ii) y is not identical with x at R**, and R** is a proper subregion of R.

The intuitive idea behind PI-3 is that, while PI-2 leaves it open that each of x and y could have a proper subregion of R at which it is identical with itself but not the other, PI-3 closes this possibility off. Given PI-3, parthood is still irreflexive. However, I have to confess that the argument for the transitivity of parthood is much harder to see. If as a matter of necessity an object never occupies a proper subregion of a region it occupies, we can prove that parthood is transitive given PI-3. But although this claim seems very reasonable, I do not have a proof for it.

Here is why this claim would give us a proof of transitivity. If we accept PI-3, then there are three rather than two conditions that must be met for x to be a part of y at a region. If condition (iii) is not met, then there is a region, R**, such that z = z at R**; but this region is a proper subregion of R, a region z also occupies. So a necessary condition on clause (iii) not being met is that an object never occupies a proper subregion of a region it occupies. Sanford (1993, p. 222) cites a literary example from Borges, in which a character reports that, "I saw the earth in the Aleph and in the earth the Aleph once more and the earth in the Aleph...." For what it is worth, on PI-2, Borge's Aleph is metaphysically possible, but on PI-3 it is not. I do not regard this as a serious advantage of PI-2 over PI-3. Question: if Borge's Aleph example is metaphysically possible, should we give up the transitivity of parthood?

Some philosophers deny that the parthood relation is asymmetric, because of the particular solution to the puzzle of material constitution they prefer. (This puzzle will be discussed momentarily.) However, these philosophers are not happy with the claim that parthood is anti-symmetric either! Frankly, I would rather have it fall out of an analysis of parthood that parthood is transitive than that it is asymmetric, especially when it already falls out of the analysis that it is anti-symmetric and irreflexive. In what follows, I will take PI-2 as the analysis of parthood, but keep in mind that PI-3 will do much of the same work.[15]

PI-2 (and PI-3) implies that parthood is extensional. The way to formulate extensionality given that parthood is identity is this: for all x and y and R, if x and y have the same parts at R, then x = y at R. We will also prove extensionality via reductio.

1. Let R be a region such that x and y have the same parts at R but x is not y. (assumption)
2. x is not identical with y at R. (simplification, premise 1)
3. Let z be a part of x and y at R. (universal instantiation, premise 1)
4. z is identical with x at R. (PI-2, premise 2)
5. z is identical with y at R. (PI-2, premise 2)
6. So x is identical with y at R. (transitivity of identity, premises 3 and 4)
7. So parthood is extensional. [premises 1, 5, 6]

[15] Thomson (1998) rejects both asymmetry and anti-symmetry.

Unlike transitivity, extensionality is fairly controversial. Many philosophers believe that the so-called puzzle of material constitution shows that parthood must not be extensional. At t-, a lump of clay is on a desk. At t, an artist manipulates the lump thereby creating a statue we will call 'Sam'. The lump still exists at t, but is not Sam, since the lump existed at a time during which Sam did not. Yet they have the same parts, and so extensionality must be false.

It is clear what the friend of PI-2 or PI-3 should say. She is already committed to some identities being 'temporary'. The example of the statue and the clay is another example of a temporary identity. In general, the puzzle of material constitution provides just another set of examples of temporary identities beyond the standard fission and fusion cases. Interestingly, most friends of temporary identity have embraced the extensionality of parthood. But why? Initially it looks like the two doctrines are logically independent of each of each other, so why do they form such a nice package of views? The friend of parthood as identity has an explanation for this: both the extensionality of parthood and the temporary identity of entities in material constitution cases stem from the same source, namely the view of parthood as identity.

Given PI-2 or PI-3, parthood is extensional. The uniqueness of composition—x and y are, at R, identical if they are composed, at R, of the same xs—follows immediately as well.

The three axioms of classical mereology (as formulated by David Lewis (1991, p. 74)) are the transitivity of parthood, the uniqueness of composition, and unrestricted composition. Composition is unrestricted just in case, whenever there are some things x1 ...xn at r1 ...rn, there is a whole made out of those things at the union of r1 ...rn. Unlike transitivity and extensionality, I do not think that unrestricted composition follows from PI-2, although it is consistent with PI-2. Suppose there are objects O1 at R1 and O2 at R2. Why would it follow from this fact that there is an object that (i) occupies the union of R1 and R2 and (ii) is identical with each of O1 and O2 at the union of R1 and R2? Perhaps there is some interesting metaphysical argument for unrestricted composition in which PI-2 plays an ineliminable role. But I wager that no such argument will be anywhere near as straightforward as the arguments for transitivity and extensionality. (I also do not think that compositional universalism follows from the doctrine that composition is identity, and so the two views are alike in this respect as well.)

In fact, as far as I can tell, PI-2 is consistent with every extant account of when composition occurs. This is a good thing. Transitivity, irreflexivity, anti-symmetry, and extensionality have a far better claim to being 'conceptually' true than any theory of when composition occurs, and PI-2 explains why this is the case.

4. Problems

We have discussed several nice features of the view that parthood is identity. But we would be remiss if we did not also discuss possible objections to the view. In what follows, I will not discuss objections to the *presuppositions* of the view, such as objections to endurantism, the spacetime framework, or temporary identity. There is a large literature on each of these topics and accordingly I will focus on objections to the view parthood is identity.

Objection 1: My parts are not identical with each other!

Here is one hand, and here is another. Given PI-2, both hands are me (at some region that I occupy). And so both hands are identical with each other (at some region that I occupy). But they are obviously distinct. And so PI-2 is false.

This argument is hard to defuse, since the intuition driving it is very powerful. The intuition pushes us to hold that there is no region at which my hands are identical with each other. PI-2 implies that there is such a region, and for this reason, it is a counter-intuitive view. Were there not things to be said in favor of the view, this objection would be decisive.

But there are things to be said in favor of the view. And, moreover, we can point out several facts that are consistent with PI-2 that may soothe the intuition. Let R be the region that I occupy. First, consider a proper subregion R1 of R that, informally, plays a role in making it true that my left hand is a part of me at R. Similarly, consider a proper subregion R2 of R that, informally, plays a role in making it true that my right hand is a part of me at R. PI-2 emphatically does *not* imply that my left hand is identical to my right *at either of R1 or R2*. Nor does PI-2 imply that my left hand is identical to my right hand at the union of R1 and R2. So there are at least three cognitively salient regions at which my right hand and my left hand are numerically distinct. There is no region other than the region occupied by the whole at which the parts become one.

It seems to me that these facts soothe the intuition that my hands are distinct sufficiently for that intuition to be trumped by the theoretical benefits the view brings to the table.

Objection 2: PI-2 Implies a Bizarre Form of Existence Monism[16]

Existence monism is the doctrine that there is exactly one concrete object, that everything concrete is one.[17] Let the Universe be that concrete object that has all other objects as parts. Let R be the region occupied by the Universe. Given PI-2, all concrete things are, at R, one.

Let's first note that strictly PI-2 does not imply this kind of existence monism, although it is consistent with it. This is because PI-2 is consistent with the non-existence of the Universe. Recall that PI-2 is neutral with respect to most theories of composition, and many of these theories will imply that there is no entity that is the Universe.

However, let us set this response aside, and grant that the Universe exists. Then there is some region at which everything is one. We are in effect revisiting objection 1, this time writ large. It is a counter-intuitive consequence of PI-2 that there is some region at which my parts are identical. It is a counter-intuitive consequence of PI-2 that there is some region at which the parts of the universe are identical. But just as there are many regions at which my parts are not identical, there are a tremendous number of regions at which the parts of the universe are not identical. You are not me at any region that is the union of some regions that we respectively occupy.

Objection 3: PI-2 Helps Itself to the Notion of a Proper Subregion and so is Circular

Consider the following objection. PI-2 (and PI-3) analyzes parthood not only in terms of identity at a region, but in terms of one region being a proper subregion of another. And to say that one region is a proper subregion of another is just to say that both are regions and that one is a proper part of the other. So far from providing an analysis of the parthood relation, PI-2 (and PI-3) presupposes the notion of parthood.

[16] Thanks to Joshua Spencer for discussion here.
[17] See Schaffer (2007) for a discussion of existence monism.

One way to respond to this objection is to argue that the parthood relation exemplified by material objects is not the same relation as the one exemplified by regions. If this is right, PI-2 would not be circular, although it would define one parthood relation partly in terms of the other. However, we need to be careful here. One reason one might have for denying that the two parthood relations are identical is that the two parthood relations have different logical forms: the one exemplified by material objects is a three-place relation one of whose relata is always a region, whereas the one enjoyed by regions is a two-place relation.[18] But this reason is not a reason available to the endurantist who accepts adverbialism, for the part relation enjoyed by material objects is a two-place relation, albeit one that is always had-at-a-region.

There are two other strategies that are promising: provide an analysis of proper subregionhood that does not appeal to any notion of parthood or provide a reductive analysis of the parthood relation obtaining between regions. The extent to which these are two strategies or merely one strategy with a different point of emphasis is not clear to me.

One analysis of proper subregionhood that might not appeal to a notion of parthood is the analysis that takes regions to be sets of spacetime points and the subregion relation to the subset relation defined on sets of spacetime points.[19] (That regions are sets of spacetime points is defended by Cartwright (1975); for criticism see Fowler (2009, chapter 1).) A more extreme response is to hold that spacetime regions are not singular unities, but rather are mere pluralities of spacetime points. On this view, 'one' region is a subregion of another just in case every point among the first plurality is among the second.

Alternatively, one might try to analyze the relation of subregionhood by appealing to the various geometrical relations obtaining between regions. I can think of no reason why the mereological properties of regions might fail to supervene on their geometrical ones. Here is one possible analysis: R1 is a proper subregion of R2 just in case R1 is zero-distance from R2 and there is some region R3 such that (i) R3 is zero distance from R2 and R3 is some greater than zero distance from R1.

[18] See McDaniel (2004) for an example of this argument.

[19] I hedge somewhat here because David Lewis (1991) argues that sets have their subsets as parts.

Objection 4: PI-2 is Inconsistent with the Claim that 'Once a Simple, Always a Simple'[20]

Aren't things without parts always things without parts? How could something that once had parts come to be something that has many parts? Put formally (and non-rhetorically), the following principle is necessarily true: for any thing x and regions R and R*, if x occupies R and x is simple at R and x occupies R*, then x is simple at R*. Yet it seems that PI-2 is inconsistent with this claim. Let R* be the region I occupy, and let R be a region occupied by one of my simple parts. Since this simple = me at R*, and I have proper parts at R*, the simple has proper parts at R*.

Since the principle that 'once a simple, always a simple' is somewhat attractive, it is fortunate that it is consistent with PI-2. The objection goes awry in exactly the place that previous objections have gone awry: it makes an illicit appeal to Leibniz's Law. True, the simple is me at R*, and I do have the property *has proper parts* at R*, but this property is a region-encoding property, and hence the inference from these truths to the claim that the simple has proper parts at R* is invalid.

Objection 5: PI-2 Rules Out the Possibility of Composite Point-Sized Objects

Consider an x that occupies a point-sized region of spacetime R. If PI-2 is true, then x is a composite object only if there is some proper subregion R at which something y is not identical with x. But R, since it is point-sized, has no proper subregions. So PI-2 rules out the possibility of point-sized composite objects.

I think this argument is sound. The question then is to what extent we should be troubled by it. PI-2 does not imply that an object is a simple at a region R only if R is a point-sized region. Occupying a point-sized region is a *sufficient* condition for being simple at that region, not a necessary condition. So PI-2 is consistent with, but does not imply, the possibility of extended simples, for whatever that is worth.

Moreover, PI-2 is consistent with the possibility of co-located but numerically distinct objects. Perhaps certain fundamental particles can 'pass

[20] Thanks to Cody Gilmore for pressing this worry.

through' each other as they travel along opposite trajectories. In this respect, PI-2 is different from an analysis of parthood according to which x is a part of y just in case x occupies a (proper or improper) subregion of a region occupied by y; this view does imply that co-located yet distinct objects are impossible.

If you believe that co-located yet distinct objects are possible, and you believe that composition is unrestricted (an assumption I will provisionally grant for the sake of argument), then there is pressure to believe that composite point-sized objects are possible. At t1, two particles of the sort capable of passing through each other approach via opposite trajectories. At t2, particles occupy the same space. At t3, the particles go their separate ways. At t2, given that composition is unrestricted, there is a composite point-sized object.

This sort of argument will be compelling to those who have a theory of composition which implies that, if there are two things co-located at t2, then there is a fusion of those things at t2. But there is a way out of the argument: deny that there are two things at t2. Perhaps the correct description of the case is that, at t2, the two particles are identical. Later, they are they are distinct once more.

Interestingly, PI-2 does not imply that in general there cannot be wholes made of co-located parts. The only kind of composite object ruled out is a point-sized one. I leave it to the reader to judge how serious a cost this.

Objection 6: This View is Crazy

Yeah, it is kind of weird. But if you don't think temporary identity is so weird that it should be dismissed out of hand, I don't get why you'd think this view should be. (If you think that temporary identity is demonstrably false, then clearly you ought to reject any view that presupposes it.) And there are many things to be said in favor of the view. It accounts for the intuition that there is an intimate connection between part and whole; it does not face the objections facing the alternative explanation, specifically, that composition is identity; it explains why parthood has the various logical properties that it has been thought to have; and other than its weirdness, it faces no devastating objections that are over and above the objections to its assumptions, most of which are staunchly defended in the literature.

3

Mereology and Modality

GABRIEL UZQUIANO

Can mereological fusions change their parts? The axioms of classical mereology do not speak directly to this question, and yet a great many philosophers who take parthood to be governed by these axioms seem to assume they cannot change their parts.[1] Curiously, dissenters tend to depart from classical mereology at least when it comes to the uniqueness of composition: no two mereological fusions ever fuse exactly the same objects.[2] I would like to argue that this is more than a remarkable coincidence; there are reasons of principle why one's adherence to classical mereology should exert some pull towards the hypothesis that fusions cannot change their parts. There is, however, no direct route from the combination of classical mereology and propositional modal logic to this hypothesis.

Why should anyone expect fusions to have their parts necessarily? One may perhaps be motivated by a suggestive model of the part–whole relation as partial identity as intimated by authors such as D. M. Armstrong and D. Baxter.[3] Identity is generally supposed not to be a source of contingency: identical objects are necessarily identical and distinct objects are necessarily distinct. Why would we expect parthood to be different in this respect?

Let me explain. The necessity of identity is a consequence of Leibniz's Law of indiscernibility of identicals and the further premise that every

[1] One reason to think this is not merely a misimpression on my part is that van Inwagen (2010) appears to be motivated by a similar state of affairs. He provides some evidence in the form of examples in footnote 1.

[2] Two examples are Fine (1999) and Thomson (1998).

[3] See Armstrong (1978, pp. 37–8) and Baxter (1988b).